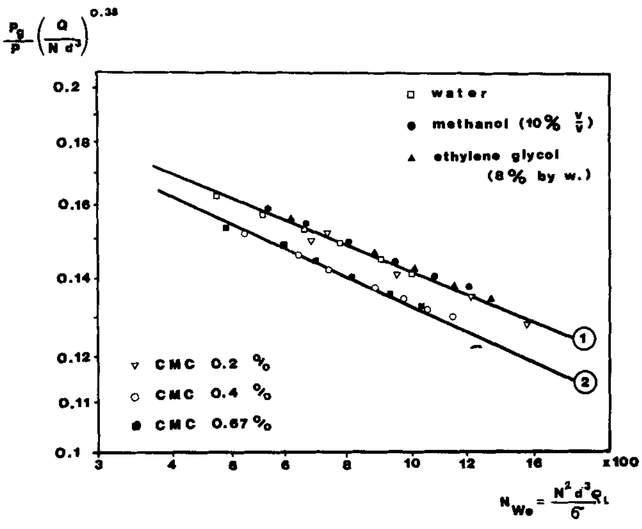

Figure 5. Variation of power function with impeller Weber number.

1) $\frac{P_g}{P} = 0.497 \left(\frac{Q}{N d^3}\right)^{-0.38} \left(\frac{N^2 d^3 \rho_L}{\sigma}\right)^{-0.18}$

Correlation coefficient = 0.985

2) $\frac{P_g}{P} = 0.514 \left(\frac{Q}{N d^3}\right)^{-0.38} \left(\frac{N^2 d^3 \rho_L}{\sigma}\right)^{-0.194}$

Correlation coefficient = 0.989

is slightly different from that for Newtonian solutions (Figure 5). In effect, the exponent n and the constant C in Equation (1) do not merely reflect geometric system parameters but also depend on the characteristics of fluids. The CMC solution (0.2%) data, however, resulted in the P_g/P ratio quite similar to the one for Newtonian solutions at the same operating conditions. This can be explained by still rather Newtonian character of the weak CMC solution.

This work is aware that the effect of isothermal expansion of gas pumped into the gas-liquid dispersion was neglected in evaluating the mixing power input data. According to Hassan and Robinson (1977), this is allowable for many practical applications.

NOTATION

- C = proportionality constant in Equation (1)
- d = impeller diameter
- N = impeller rotational speed
- N_p = power number, $P/\rho_L N^3 d^5$
- P = mechanical agitation power in ungassed liquid
- P_g = mechanical agitation power in gassed liquid
- Q = volumetric gas sparging rate
- ρ_L = mass density of liquid
- σ = air-liquid surface tension

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The Effect of the Array of Disks on Mass Transfer Rates to the Tube Walls

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In a recent publication (Koncar-Djurdjevic and Dudukovic, 1977), we have discussed the effect of single stationary objects placed in the fluid stream on mass transfer rates to the walls of a coaxial cylindrical tube. The results obtained indicated very complex interactions

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of the wake formed behind the object with the boundary layer at the tube walls. These interactions result in two maxima and one minimum of the curve for the local Sherwood number as a function of distance from the object along the tube. The aim of the present investigation is to examine the effect of the next object, placed coaxially behind the first one on the wake behind the former one and on the wake interaction with the boundary layer at the tube walls. This experimental information is of interest from the fundamental point of view, but the obtained results could also be important for practical applications.

The local Sherwood number in the system under study is a function of the distance along the tube, distance between the objects, Reynolds number, as well as of the ratio of the object and tube diameters. To investigate the dependence of the local Sherwood number on all of the above mentioned variables would call for a very large number of experiments. However, it can be expected that the various types of interactions of the wakes behind the first and second object can be classified in terms of characteristic distances between the objects. We decided to determine indirectly these characteristic distances and their dependences on the Reynolds number and on the ratio of object to tube diameter by relying on the analogy between momentum and mass transport and by measuring the flow drag. Flow drag measurements are more rapid and more readily done than the determination of local mass transfer coefficients. This approach rests on the assumption that the drag for two disks located in a tube very far apart from each other is twice the drag when they are placed next to each other, while for some intermediate distance the drag will exhibit either an extremum or at least an inflection point when plotted as a drag coefficient against the distance between the disks. This assumption was confirmed by our experimental results. The distance between the disks at which the drag shows an extremum or inflection point was named characteristic distance. After the dependence of the characteristic distances on Reynolds number and tube to disk diameter ratio have been determined, one can experimentally evaluate the dependence of local Sherwood number on the distance along the tube for a particular Reynolds number and for the corresponding characteristic distance and object to tube diameter ratio. Herewith, the same phenomena would be present for the characteristic distances at different Reynolds numbers and different ratios of object and tube diameters. The values of local Sherwood numbers would differ, but their dependences on Reynolds number and on the ratio of object and tube diameter were previously investigated (Koncar-Djurdjevic and Dudukovic, 1977).

APPARATUS AND EXPERIMENTAL METHOD

Two disks of the same diameter were placed coaxially in a cylindric tube 60 mm in diameter. The first disk was placed 35 tube diam from the entrance. Disks were placed on the shaft with a thread 5 mm thick and 200 mm long, enabling mounting of the second disk on different distances from the first one. The first disk was 2 mm thick and the second one 3 mm. Both were tapered around the circumference at a 45 deg angle in the downstream direction producing the effect analogous to that of an extremely thin disk. The shaft with the mounted disks was fastened coaxially in the tube, just behind the first disk as well as at the end of the shaft with three bolts of 3 mm, which were set at a 120 deg angle, perpendicular to the flow direction. The fluid was tap water.

The pressure drop was measured at the ends of the operating tube by U manometers filled with mercury, chloroform, or methylbenzoate.

For determination of local coefficients of mass transfer we applied the adsorption method developed by S. Koncar-Djurdjevic (1949, 1953, 1956). This method was applied in the same way and under the same conditions as reported in our previous work (Koncar-Djurdjevic and Dudukovic, 1977) and consists of measuring the amount of methylene blue deposited from water solution on silica gel which covers the wall over a period of time during which the irreversible adsorption is completely mass transfer controlled.

RESULTS

Pressure drops in the operating tube were measured for different Reynolds numbers, different ratios of disks and tube diameters, and different distances between disks.

The drag coefficient can be calculated from the pressure drop in the fluid where from the overall measured Δp the calculated Δp for an empty tube is subtracted as shown by the equation given by Dudukovic (1978):

$$C_D^* = \frac{(\Delta p/\gamma)(d/d_p)^2}{w^2/2g} \tag{1}$$

Experiments were carried out for six ratios of d_p/d of 0.500, 0.667, 0.750, 0.833, and 0.917. Typical results for a single ratio of disk to tube diameter of $d_p/d = 0.667$ and $d_p/d = 0.750$ are given in Figures 1*a* and *b*. Curves in Figure 1 represent the dependence of the drag coefficient C_D^* on Reynolds number Re_p which is based on disk diameter. Each curve corresponds to a single distance between the disks expressed dimensionlessly as l/d_p. It is observed that the drag coefficient in this range of Reynolds numbers is not a strong function of Reynolds number. This could be expected, as in this range of Reynolds numbers disk drag coefficient in an infinite fluid is also independent of Reynolds number according to the results of Wieselsberger (1922) and Roos and Willmarth (1971).

The dependence of the drag coefficient C_D^* on dimensionless distance between the disks l/d_p is given in Figure 2 for the constant Reynolds number of $Re_p = 3.16 \cdot 10^4$ and for different disk to tube diameter ratios. The same type of behavior is observed at other Reynolds numbers due to the nearly constant value of the disk drag coefficient. The curves in Figure 2 have a few common characteristics. All of them approach the value of the drag coefficient for one disk when the distance between disks goes to zero and take twice that value when the distance between disks becomes large enough. The appearance of a local minimum is observed in each case.

Experiments for determination of local mass transfer coefficients were carried out for the ratio of diameters $d_p/d = 0.667$ and Reynolds number $Re = 4.74 \cdot 10^4$ based on tube diameter ($Re_p = 3.16 \cdot 10^4$). These mass transfer experiments were done using very small distances between disks, distances in the area of the drag minimum (Figure 2), at distances in the area of inflection point (Figure 2), and at very large distances. When the adsorption of methylene blue on silica gel is completely mass transfer controlled, as is the case in the present study, the local mass transfer coefficient is cal-

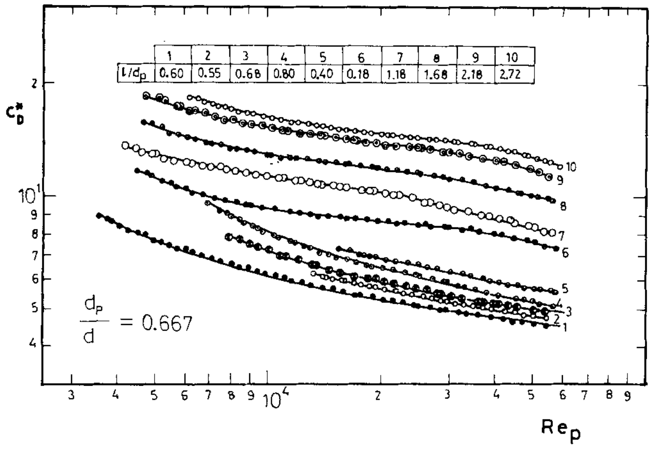

Fig. 1a. Drag coefficient for two disks as a function of Reynolds number (d_p/d = 0.667).

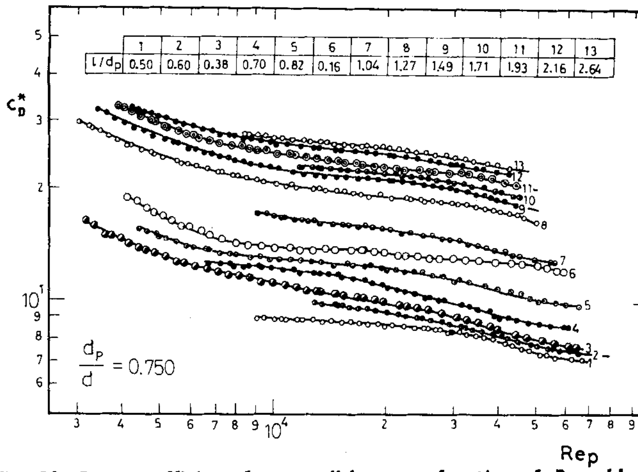

Fig. 1b. Drag coefficient for two disks as a function of Reynolds number (d_p/d = 0.750).

culated from

$$k = \frac{C_p}{\theta \cdot c_o} \tag{2}$$

The procedure is described in our previous publications (Končar-Djurdjević and Duduković, 1977; Duduković 1978).

The results are presented as the ratio of the local Sherwood number and the Sherwood number far downstream from the disks, that is, Sherwood number which characterizes an undisturbed stream in the tube. The dependence of this ratio Sh/Sh_∞ on the distance along the tube x, where $x = 0$ is a normal projection of the center of first disk on the tube wall, is given in Figures 3 to 6. The positive direction of the x axis is in the direction of fluid flow.

DISCUSSION

The curve in Figure 3 represents the dependence of dimensionless local Sherwood number Sh/Sh_∞ on the distance along the tube for small distances between the disks. The shape of the curve agrees completely with the curves obtained for one disk (Končar-Djurdjević and Duduković, 1977), which indicates that two disks at small distances from each other behave essentially as one. The first local maximum results from two effects: the narrowing of the effective cross section available for

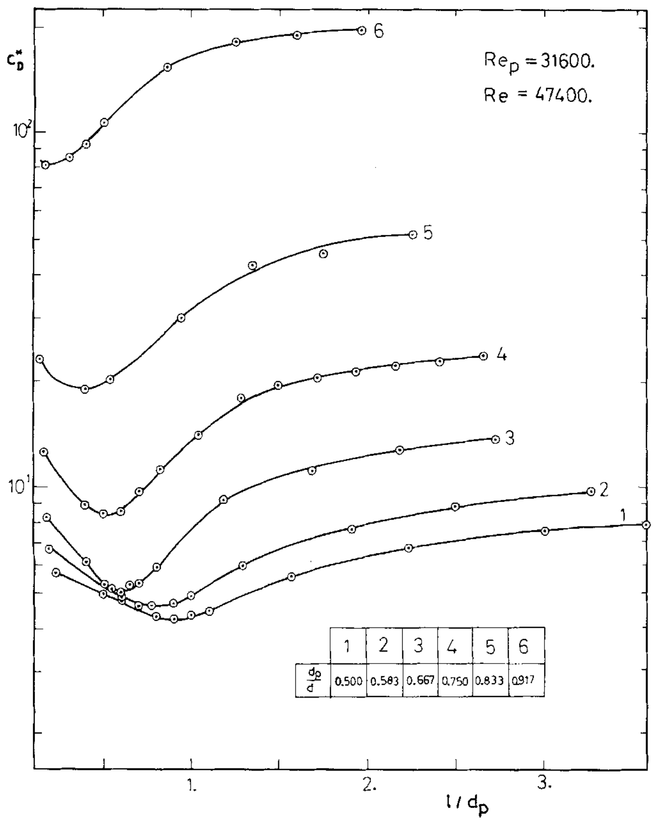

Fig. 2. Drag coefficient for two disks as a function of the separating distance (Re_p = 31 600).

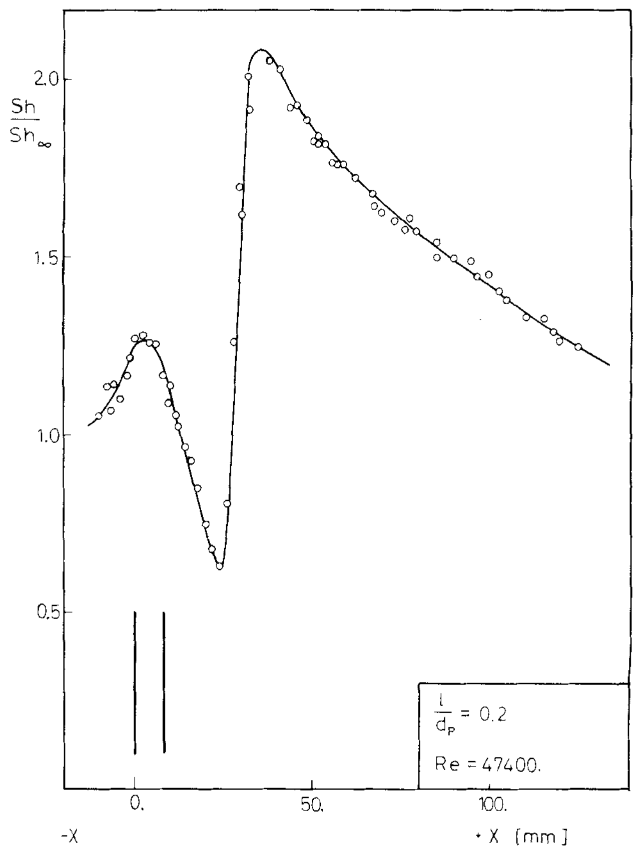

Fig. 3. Local Sherwood number as a function of distance (l/d_p = 0.2, Re = 47 400).

flow and the velocity component perpendicular to the tube wall which is imparted to the fluid. Both effects lead to the decrease of the thickness of the boundary layer at the tube wall.

The minimum and the second local maximum are the results of wake formation and its interaction with the boundary layer at the wall. Both disks form only one wake behind the second disk. The minimum is due to the fluid being sucked into the wake, causing an increase of the boundary-layer thickness at the tube wall. Owing to the instability and separation of the wake, the intensity of turbulent pulsations increases in the area behind the wake causing fluid elements to penetrate into the diffusion sublayer which leads to a rapid increase of the local Sherwood number and to the second maximum.

The results shown in Figure 4 are obtained for the distance between the disks for which the minimum of the curve $C_D^* = f(l/d_p)$ is observed in Figure 2. The curve retains the basic shape characteristic for one disk or for two disks at very small distances but with one important difference. The minimum of the curve is now immediately in front of the second disk. This means that the first disk forms its own wake, but its effective length is affected by the presence of the second disk, causing the minimum and second maximum to be closer to the first disk than in the case of a single disk. The shape of the curves also indicates that there is now no new independent wake being formed behind the second disk owing to the lack of a second minimum. The formation of the second wake seems to be disturbed by the existence and arrival of the first one. This is due to fluid suction into the first wake just in front of the second disk as well as to the high turbulence in the area behind the first wake.

For a greater distance between disks (Figure 5) corresponding to the inflection point on curve $C_D^* = f(l/d_p)$ in Figure 2, it is observed that the second disk does not influence any more the length of the first wake, since the minimum and maxima appear at same positions as in the case of a single disk. The formation of the second wake is still disturbed.

In the case of even larger distances (Figure 6), each disk behaves nearly independently. The local Sherwood numbers around the second disk are greater in their values because the intensity of turbulence pulsations is still higher owing to the presence and instability of the first wake.

The characteristic minimum on the curves representing the dependence $C_D^* = f(l/d_p)$ in Figure 2 results from the fact that two disks separated by a distance below certain critical distance behave similarly to the cylinder of the corresponding length. Comparison of drag coefficients of two disks and that of the cylinder of the corresponding length is given in Figure 7 for $d_p/d = 0.667$ and at $Re_p = 31\,600$. Up to the distances corresponding to the minimum of the curve $C_D^* = f(l/d_p)$ on Figure 2, the agreement is very good, proving our supposition. The values of drag coefficients for the cylinder are only slightly higher, being the result of the additional skin friction. At greater distances, each disk starts to act separately. Consequently, the drag is significantly higher than that of the cylinder of the corresponding length (point 3 at $l/d_p = 1.20$).

In this preliminary study we have demonstrated that the drag coefficient C_D^* for a pair of disks is a strong

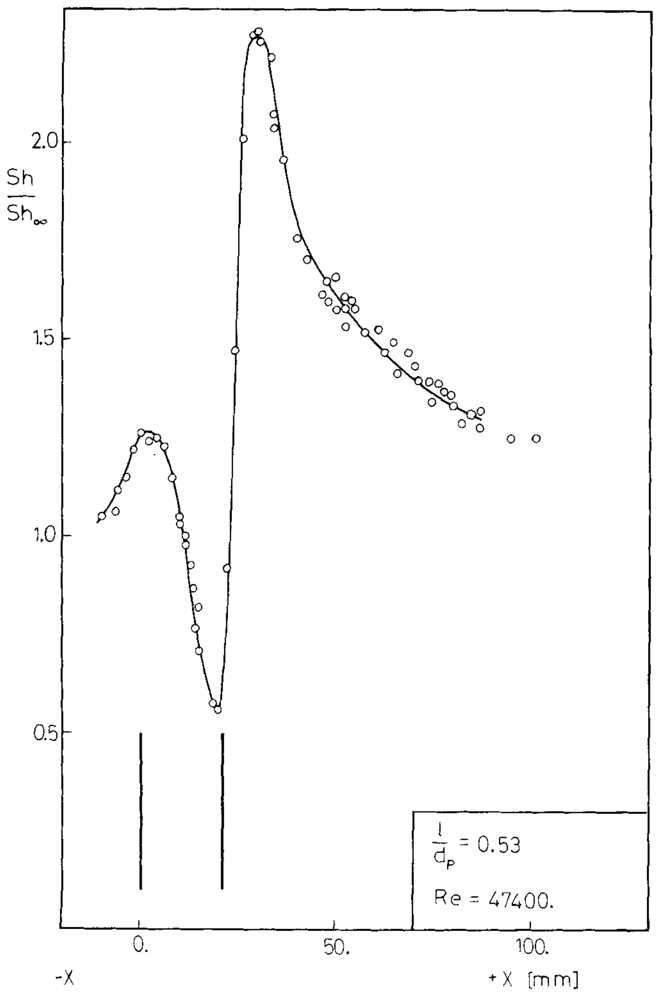

Fig. 4. Local Sherwood number as a function of distance ($l/d_p = 0.53$, $Re = 47\,400$).

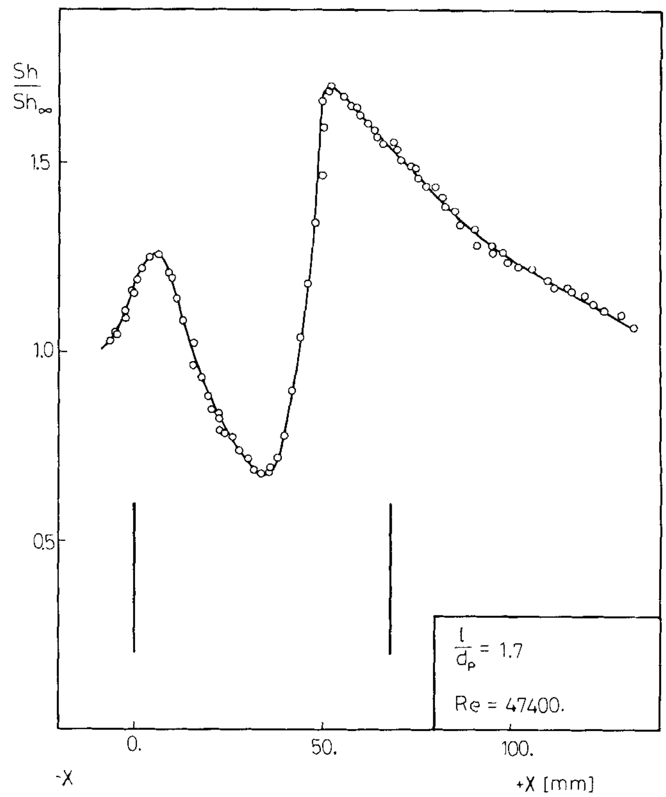

Fig. 5. Local Sherwood number as a function of distance ($l/d_p = 1.7$, $Re = 47\,400$).

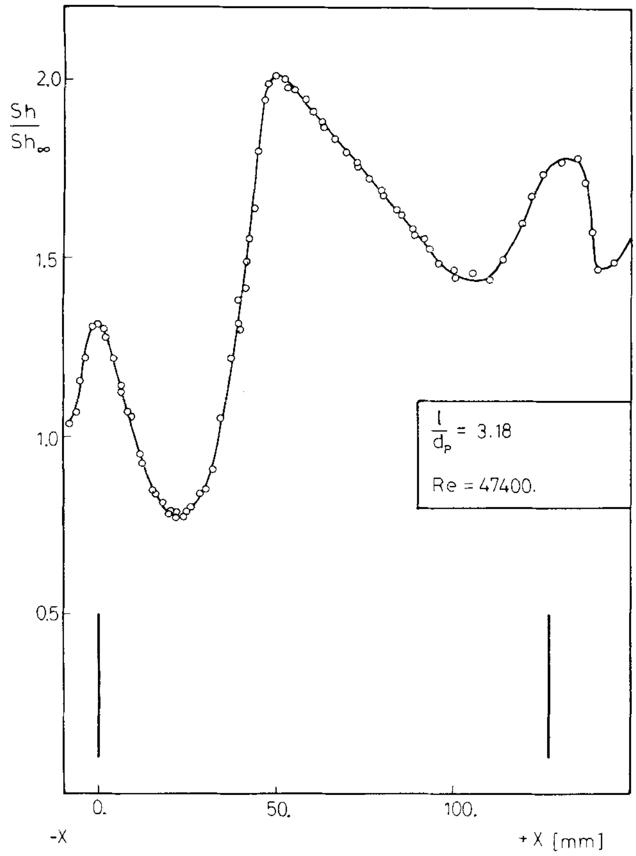

Fig. 6. Local Sherwood number as a function of distance (l/d_p = 3.18, Re = 47 400).

function of the distance between the disks (l/d_p) and of the disk to tube diameter ratio (d_p/d) while it is relatively weak function of Reynolds number for the range 10^3 to 10^5. The drag coefficient is always minimal at a certain critical distance between the disks at a given d_p/d. Up to that distance, the values of the drag coefficient for the two disks are very close to those for a cylinder of a corresponding length. The slightly lower values for the two disks may be caused by the large recirculation cell formed in between them, while additional skin friction exists in the case of a cylinder. The local Sherwood number as a function of position exhibits the same characteristic shape with two local maxima and one minimum, except when the disks far apart and the drag corresponds to the drag on two disks. The distances between the extrema and the position of the minimum and second maximum as well as their magnitude depend on the distance between disks. For distances less than the critical distance at which the C_D^* has a minimum, the local minimum appears behind the second disk. At distances corresponding to the critical distance at which C_D^* is minimum, the minimum in local Sherwood number immediately precedes the second disk, while the second local maximum appears directly behind the second disk. At distances corresponding to the inflection point on the $C_D^* = f(l/d_p)$ curve, both the minimum and second maximum appear in front of the second disk. At large distances, the Sherwood number curve for a single disk is repeated twice, but higher average values are attained. At these large distances, it appears that by placing an array of disks, an average Sherwood number 60 to 80% higher than in an empty tube can be maintained. Under these conditions the overall pressure drop is increased ten to twenty times with respect to the pressure drop in an empty tube. Further work on quantification of the above described effects is in progress.

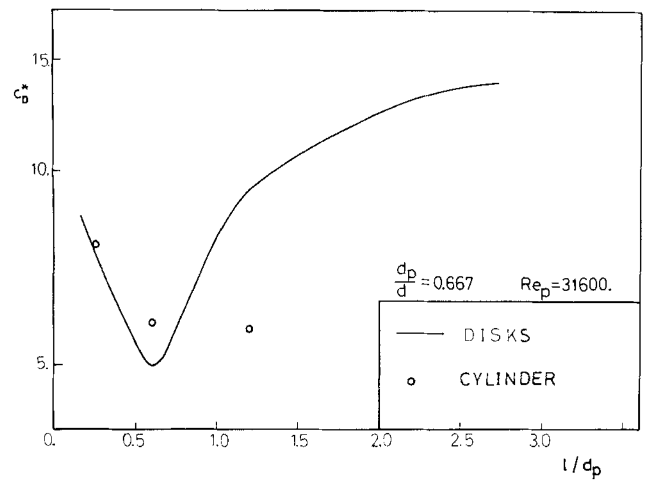

Fig. 7. Comparison of the drag coefficient for two disks with that for a cylinder (d_p/d = 0.667, Re_p = 31 600).

NOTATION

C_D^* = disk drag coefficient
c_o = bulk methylene blue concentration in the fluid
C_p = surface concentration of methylene blue on the silica gel
d = tube diameter
d_p = disk diameter
g = acceleration due to gravity
k = mass transfer coefficient
l = distance between the disks
p = pressure
Re = Reynolds number based on tube diameter
Re_p = Reynolds number based on disk diameter
Sh = local Sherwood number
Sh_∞ = local Sherwood number characteristic for undisturbed stream in the tube
w = mean velocity of fluid in the tube
x = distance along the tube
γ = fluid specific weight
θ = stationary adsorption time

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